Flight Dynamics & Control

Eigenvalue-Eigenvector Assignment

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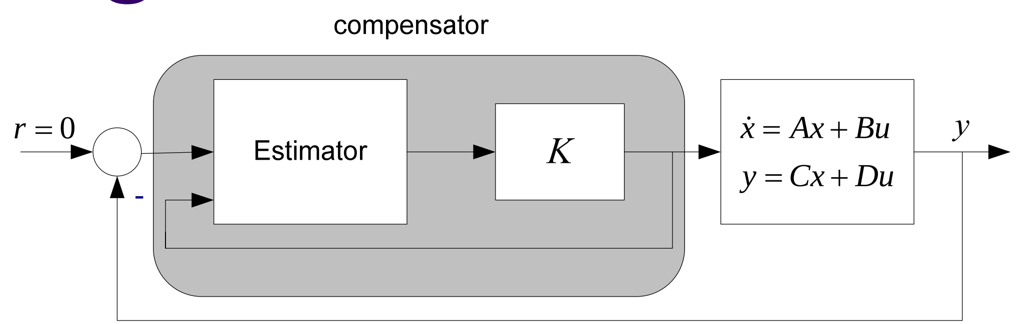
Outline

- State Feedback
  - Pole placement revisited
  - Eigenvector assignment
- Example: F-16
- Example: L1011

State Space Design



Consider the usual linear plant

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

The feedback stabilization problem: find an output feedback control that shapes the transient response of the closed loop to meet prescribed objectives.

This will be done in three steps:

1) Design a state feedback control, $u = Kx$

2) Design a state estimator, that generates state estimates $\hat{x}(t)$ from available information, i.e., $u(\tau), y(\tau) \quad \tau \in [0,t)$

3) Implement the composite controller, $u(t) = \hat{x}(t)$

Pole Assignment Problem

$$\dot{x} = Ax + Bu, \quad (A, B) \text{ completely controllable, } \operatorname{rank} B = m$$

Pole assignment problem: Given a self conjugate set of scalars $\{\lambda_1, \ldots, \lambda_n\}$ and vectors $\{v_1, \ldots, v_n\}$ find a real $m \times n$ matrix K such that the eigenvalues and eigenvectors of $(A + BK)$ are precisely the given sets.

Theorem (Wonham, 1967): The system is controllable if and only if for every self-conjugate set of scalars $\{\lambda_1, \ldots, \lambda_n\}$ there exists a real $m \times n$ matrix K such that $(A + BK)$ has $\{\lambda_1, \ldots, \lambda_n\}$ as its eigenvalues.

Some Definitions

Define the matrices

$$S_\lambda := [\lambda I - A \mid B]$$

$$R_\lambda := \{\text{columns form a basis for } \ker[S_\lambda]\} = \begin{bmatrix} N_\lambda \\ M_\lambda \end{bmatrix},\ N_\lambda \in R^{n\times k},\ M_\lambda \in R^{m\times k}$$

Note:

$$\text{controllability} \Rightarrow \dim\ker\left[S_\lambda\right] = n$$

$$\operatorname{rank} B = m \Rightarrow \operatorname{rank} N_\lambda = m$$

$$N_{\bar{\lambda}} = \bar{N}_\lambda$$

Main Result on Pole Assignment

Theorem (Moore 1976): Let $\{\lambda_i, i=1,\ldots,n\}$ be a set of self-conjugate scalars. There exists a real $m \times n$ matrix K such that

$$(A+BK)v_i = \lambda_i v_i,\ i=1,\ldots,n$$

if and only if

1) $\{v_i, i=1,\ldots,n\}$ are linearly independent

2) $v_i = \overline{v}_j$, when $\lambda_i = \overline{\lambda}_j$

3) $v_i \in \operatorname{Im} N_{\lambda_i}$

Also, if K exists and $\operatorname{rank} B = m$ then K is unique.

Proof: Necessity

$$
\begin{aligned}
&(A+BK)v_i = \lambda_i v_i \\
&\quad \Rightarrow (\lambda_i I - A)v_i = -BKv_i \\
&\quad \Rightarrow \begin{bmatrix} \lambda_i I - A & B \end{bmatrix} \begin{bmatrix} v_i \\ -Kv_i \end{bmatrix} = 0
\end{aligned}
$$

$$
S_{\lambda_i} \begin{bmatrix} v_i \\ -Kv_i \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} v_i \\ -Kv_i \end{bmatrix} \in \operatorname{Im} R_{\lambda_i} \Rightarrow v_i \in \operatorname{Im} N_{\lambda_i}
$$

Proof: Sufficiency, 1

assume the set $\{v_i, i = 1, \ldots, n\}$ satisifies 1), 2), 3)

3) $\Rightarrow$ there exists $z_i \in \mathrm{C}^{\mathrm{k}}$ such that $v_i = N_{\lambda_i} z_i$

By definition $S_{\lambda_i} R_{\lambda_i} = 0 \Rightarrow$

$$(\lambda_i I - A) N_{\lambda_i} + BM_{\lambda_i} = 0$$

$$(\lambda_i I - A) N_{\lambda_i} z_i + BM_{\lambda_i} z_i = 0$$

Suppose, K can be chosen such that $M_{\lambda_i} z_i = -Kv_i$

Then, it would follow that $\left[\lambda_i I - (A + BK)\right] v_i = 0.$

Thus, real K is to be chosen so that

$$\boxed{K\begin{bmatrix} v_1 & \cdots & v_n \end{bmatrix} = \begin{bmatrix} -M_{\lambda_1} z_1 & \cdots & -M_{\lambda_n} z_n \end{bmatrix}}$$

Proof: Sufficiency, 2

Assumption 1) implies that this is always possible.

If the λ_i's are real, we simply compute

$$K = \begin{bmatrix} -M_{\lambda_1} z_1 & \cdots & -M_{\lambda_n} z_n \end{bmatrix} \begin{bmatrix} v_1 & \cdots & v_n \end{bmatrix}^{-1}$$

If some λ_i's are complex, proceed as follows for each complex

conjugate pair. Suppose $\lambda_1 = \overline{\lambda}_2$ so that from 2) $v_1 = \overline{v}_2 \Rightarrow z_1 = \overline{z}_2$.

For simplicity suppose all other eigenvalues are real. Define

$$w_i := M_{\lambda_i} z_i ,$$

and use the notion, for any complex quantity $a = a_R + ja_I$. Then

Proof: Sufficiency 3

$$K\begin{bmatrix} v_1 & \cdots & v_n \end{bmatrix} = \begin{bmatrix} -M_{\lambda_1} z_1 & \cdots & -M_{\lambda_n} z_n \end{bmatrix} \Rightarrow$$

$$K\begin{bmatrix} v_{1R} + jv_{1I} & v_{1R} - jv_{1I} & v_3 & \cdots & v_n \end{bmatrix} = \begin{bmatrix} w_{1R} + jw_{1I} & w_{1R} - jw_{1I} & -M_{\lambda_3} z_3 & \cdots & -M_{\lambda_n} z_n \end{bmatrix}$$

post multiply by the nonsingular matrix

$$\begin{bmatrix} \begin{matrix} 1/2 & -j1/2 \\ 1/2 & j1/2 \end{matrix} & 0 \\ 0 & I \end{bmatrix}$$

to obtain

$$K\begin{bmatrix} v_{1R} & v_{1I} & v_3 & \cdots & v_n \end{bmatrix} = \begin{bmatrix} w_{1R} & w_{1I} & -M_{\lambda_3} z_3 & \cdots & -M_{\lambda_n} z_n \end{bmatrix}$$

$$\boxed{K = \begin{bmatrix} w_{1R} & w_{1I} & -M_{\lambda_3} z_3 & \cdots & -M_{\lambda_n} z_n \end{bmatrix}\begin{bmatrix} v_{1R} & v_{1I} & v_3 & \cdots & v_n \end{bmatrix}^{-1}}$$

Finally, since a fixed eigenstructure uniquely defines $(A + BK)$, it can be proved that K is unique when $\operatorname{rank} B = m$.

Geometry

- A subset $\mathscr{S}$ of the linear space (over field $\mathbb{F}$) $\mathscr{X}$ is a linear subspace of $\mathscr{X}$ if $\forall x_1, x_2 \in \mathscr{S}$ and $\forall c_1, c_2 \in \mathbb{F}, c_1 x_1 + c_2 x_x \in \mathscr{S}$
- If $x_i \in \mathscr{X}$ $(i = 1, \ldots, k)$, then $\operatorname{span}\{x_1, \ldots, x_k\}$ is a subspace of $\mathscr{X}$.
- $\mathscr{R}, \mathscr{S} \subset \mathscr{X}$ then

$$\mathscr{R} + \mathscr{S} = \left\{ r + s \,\middle|\, r \in \mathscr{R}, s \in \mathscr{S} \right\}$$

$$\mathscr{R} \cap \mathscr{S} = \left\{ x \,\middle|\, x \in \mathscr{R} \;\&\; x \in \mathscr{S} \right\}$$

- Two subspaces $\mathscr{R}, \mathscr{S}$ are independent if $\mathscr{R} \cap \mathscr{S} = 0$

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Geometry 2

- If $\mathscr{R}_i,\ i = 1,\ldots,k$ are independent subspaces, then the sum

$$\mathscr{R} = \mathscr{R}_1 + \cdots + \mathscr{R}_k$$

is called an indirect sum and may be written

$$\mathscr{R} = \mathscr{R}_1 \oplus \cdots \oplus \mathscr{R}_k$$

The symbol $\oplus$ presuposes independence.

- Let $\mathscr{X} = \mathscr{R} \oplus \mathscr{S}$. For each $x \in \mathscr{X}$, there are unique $r \in \mathscr{R}, s \in \mathscr{S}$ so that $x = r + s$. This implies a unique function $x \mapsto r$ called the projection on $\mathscr{R}$ along $\mathscr{S}$.

Geometry 3

- The projection is a linear map $Q : \mathscr{X} \to \mathscr{X}$, such that

$\operatorname{Im} Q = \mathscr{R}$ and $\ker Q = \mathscr{S}$, and

$$\mathscr{X} = Q\mathscr{X} \oplus (I - Q)\mathscr{X}$$

- Note that $(I - Q)$ is the projection on $\mathscr{S}$ along $\mathscr{R}$. Thus,

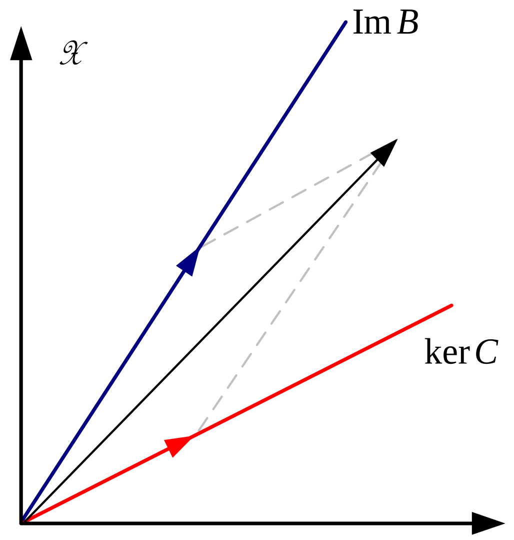
$$Q(I - Q) = 0 \Leftrightarrow Q^2 = Q$$

- Conversly, for any map $Q : \mathscr{X} \to \mathscr{X}$ such that $Q^2 = Q$

$$\mathscr{X} = \operatorname{Im} Q \oplus \ker Q$$

i.e., Q is the projection on $\operatorname{Im} Q$ along $\ker Q$.

Geometry 4



$$Q = B(CB)^{-1}C$$

$$Q^2 = B(CB)^{-1}CB(CB)^{-1}C = B(CB)^{-1}C = Q$$

$$\operatorname{Im} Q = \operatorname{Im} B \quad \ker Q = \ker C \quad X = \operatorname{Im} B \oplus \ker C$$

$$\boxed{\begin{array}{l} B(CB)^{-1}C \text{ is the projection on } \operatorname{Im} B \text{ along } \ker C \\ \left(I - B(CB)^{-1}C\right) \text{ is the projection on } \ker C \text{ along } \operatorname{Im} B \end{array}}$$

Example: F-16

landing approach

$$\begin{bmatrix}\dot{u}\\ \dot{\alpha}\\ \dot{q}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}-0.507 & -3.861 & 0 & -32.17\\ -0.00117 & -0.5164 & 1 & 0\\ -0.000129 & 1.4168 & -0.4932 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}u\\ \alpha\\ q\\ \theta\end{bmatrix}+\begin{bmatrix}0\\ -0.0717\\ -1.645\\ 0\end{bmatrix}\delta_E$$

$$y=\begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}u\\ \alpha\\ q\\ \theta\end{bmatrix}$$

$$\text{phugoid: } \lambda=-0.0438167\pm j0.206461 \quad h=\begin{bmatrix}0.999978\\ 0.000484\\ 0.001343\\ -0.000272\end{bmatrix}\pm j\begin{bmatrix}0\\ 0.0002676\\ 0.0002264\\ -0.0064497\end{bmatrix}$$

$$\text{short period: } \lambda=-1.7036,\ 0.730937 \quad h=\begin{bmatrix}-0.994287\\ -0.063373\\ 0.074073\\ -0.043481\end{bmatrix},\begin{bmatrix}0.999508\\ -0.014171\\ -0.016507\\ -0.022584\end{bmatrix}$$

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Example: F-16 state feedback

Desired poles -

short period: $\lambda_{1,2} = -1.25 \pm j2.16506$

phugoid: $\lambda_{3,4} = -0.01 \pm j0.0994987$

$$\operatorname{Re} R_{\lambda_1} = \begin{bmatrix} -0.942209 \\ -0.049274 \\ -0.088225 \\ -0.029211 \\ \hline 0.260982 \end{bmatrix}, \quad \operatorname{Im} R_{\lambda_1} = \begin{bmatrix} 0 \\ -0.048293 \\ 0.135264 \\ -0.057615 \\ \hline 0.095482 \end{bmatrix}$$

$$\operatorname{Re} R_{\lambda_3} = \begin{bmatrix} -0.999992 \\ 0.001453 \\ -0.000320 \\ 0.001090 \\ \hline -0.001428 \end{bmatrix}, \quad \operatorname{Im} R_{\lambda_3} = \begin{bmatrix} 0 \\ 0.000117 \\ -0.000077 \\ -0.003107 \\ \hline -0.000104 \end{bmatrix}$$

$$K = \begin{bmatrix} 0.004076 & 3.87578 & 0.718424 & 0.095189 \end{bmatrix}$$

Example: F-16 Rynaski "robust observer"

"place observer poles at LHP plant zeros, remainder are placed arbitrarily"

$$\lambda = 0, -0.04231, -0.5865, -1$$

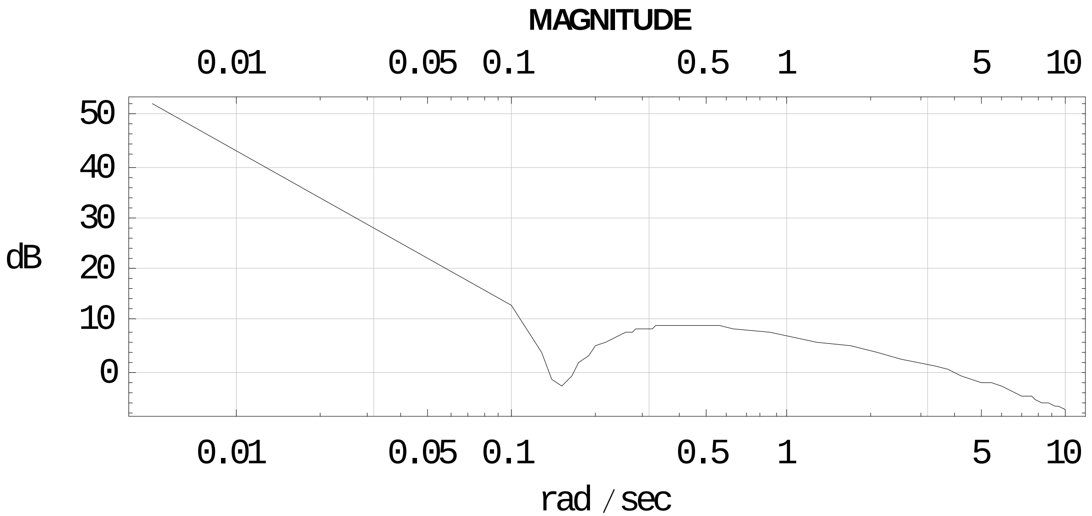
$$R_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.707107 \\ \hline 0.707107 \end{bmatrix}, R_2 = \begin{bmatrix} 0.000934 \\ -0.006733 \\ 0.000293 \\ 0.710515 \\ \hline 0.703649 \end{bmatrix}, R_3 = \begin{bmatrix} 0.001445 \\ 0.665243 \\ -0.028975 \\ 0.079296 \\ \hline 0.741837 \end{bmatrix}, R_4 = \begin{bmatrix} 0.000863 \\ 0.73183 \\ -0.247431 \\ 0.027934 \\ \hline 0.634367 \end{bmatrix}$$

$$L^T = \begin{bmatrix} 0.168343 & -1.02106 & -0.56851 & -1 \end{bmatrix}$$

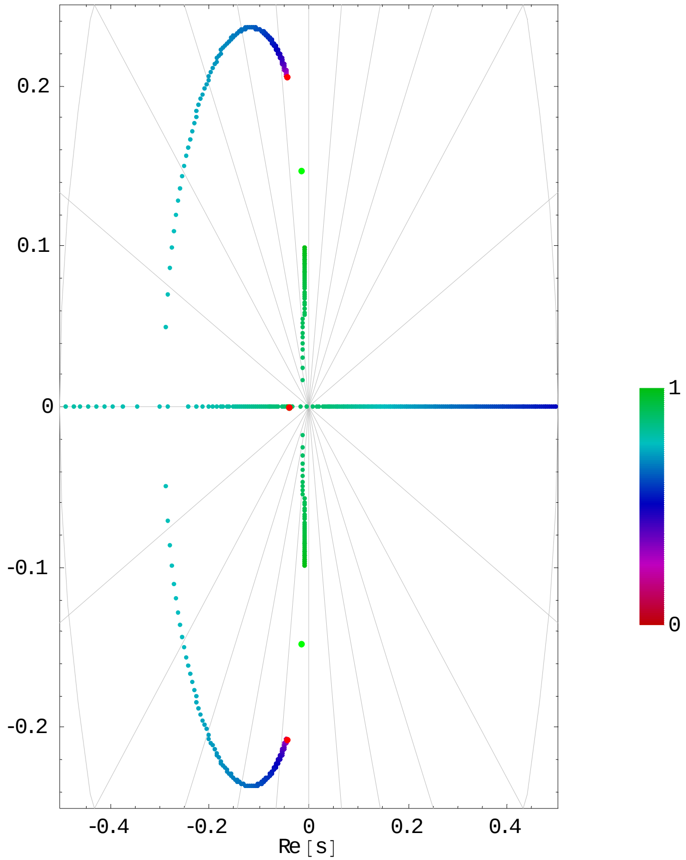
Example: F-16

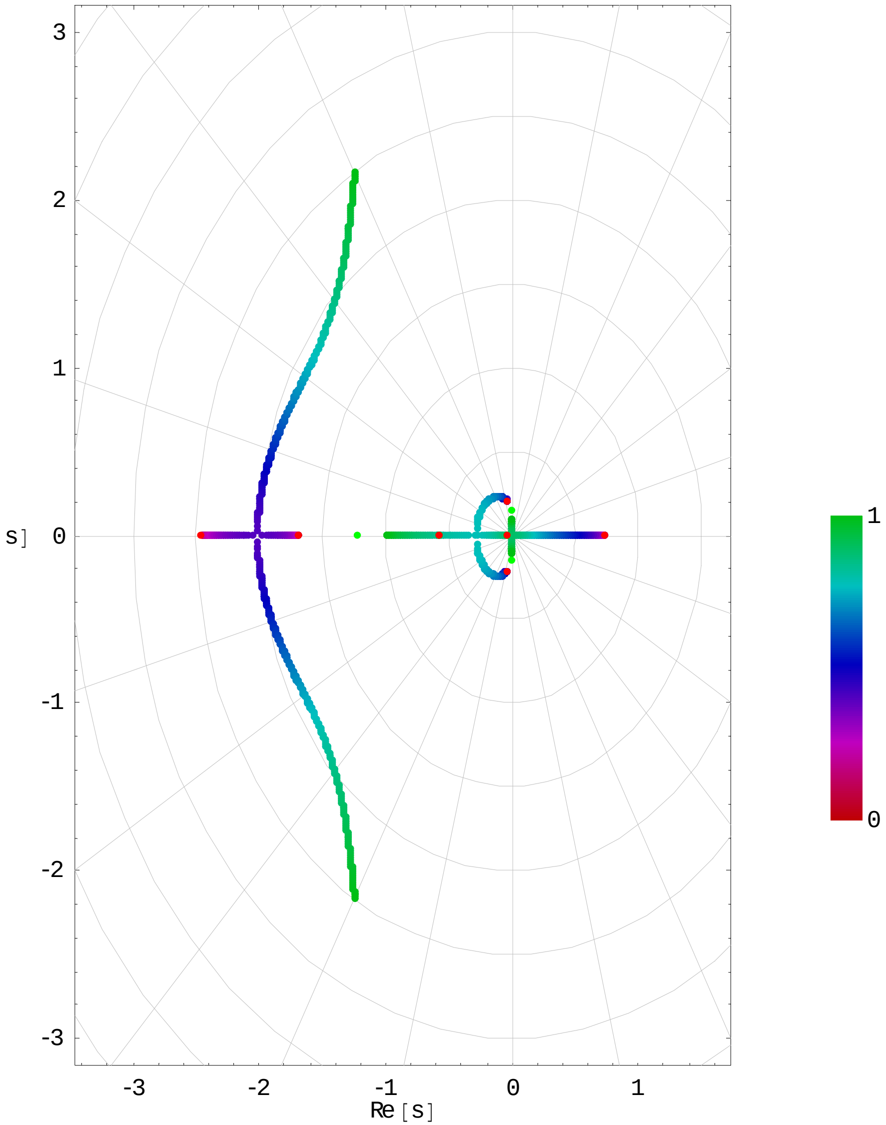
$$G_p(s) = 1.645\frac{s(s+0.0423101)(s+0.586543)}{(s-0.730937)(s+1.7036)(s^2+0.0876334s+0.044546)}$$

$$G_c(s) = 4.46035\frac{(s+2.45962)(s+0.0148335\pm j0.147508)}{s(s+0.423102)(s+0.586577)(s+2.45962)}$$



Example F-16





F-16 CCV

- The first YF-16 (72-1567) was rebuilt in December 1975 to become the USAF Flight Dynamics Laboratory's Control Configured Vehicle (CCV). CCV aircraft have independent or "decoupled" flight control surfaces, which make it possible to maneuver in one plane without movement in another--for example, turning without having to bank.
- The CCV YF-16 was fitted with twin vertical canards added underneath the air intake, and flight controls were modified to permit use of wing trailing edge flaperons acting in combination with the all moving stabilator.
- The YF-16/CCV flew for the first time on March 16, 1976, piloted by David J. Thigpen. On June 24, 1976, it was seriously damaged in a crash landing after its engine failed during a landing approach. The aircraft was repaired and its flight test program was resumed. The last flight of the YF-16/CCV was on June 31, 1977, after 87 sorties and 125 air hours had been logged.

F-16 AFTI

- The Flight Dynamics Laboratory of the Air Force Systems Command sponsored an Advanced Fighter Technology Integration (AFTI) program. In 1979, General Dynamics was awarded a contract to convert the fifth FSD F-16A (75-0750) into an AFTI aircraft. It capitalized on the experience gained with the CCV (Control Configured Vehicle) F-16 (72-1567).
- The AFTI F-16 was fitted with twin canard surfaces mounted below the air intake, these surfaces having been taken from the CCV/F-16. It had a full-authority triplex Digital Flight Control System (DFCS) and an Automated Maneuvering Attack System (AMAS). This system provides six independent degrees of freedom.
- It was designed to be fault tolerant, so that no single failure should affect correct operation. In the event of a second fault, the system reverts to a standby condition which will permit safe flight to continue. The system incorporates an analog backup flight-control system.
- The AFTI first took to the air July 10. Phase I testing involved the demonstration of direct translational maneuvering capability. Phase II testing (1984-87) involved F-16C-standard avionics with AMAS. The AMAS enabled the AFTI/F-16 to translate in all three axes at a constant angle of attack and to be pointed up to six degrees off the flight vector.
- In recent years, the AFTI/F-16 became associated with close air support (CAS) studies, some of them conducted by NASA. These studies began in 1991.

Multimode, High Maneuverability Flight Control

- Sobel & Shapiro, 1985
- Longitudinal
  - Pitch pointing/ vertical translation - command the pitch angle without a change in flight path angle
  - Direct lift – command normal acceleration (or flight path angle rate) without affecting angle of attack
- Lateral
  - Yaw pointing/ lateral translation– decouple the lateral directional response from roll (bank) angle and rate and yaw rate
  - Direct sideforce – command lateral acceleration without a change in sideslip angle

Example: F-16 CCV

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -0.8693 & 43.223 & -17.251 & -1.5766 \\ 0 & 0.9933 & -1.341 & -0.1689 & -0.2518 \\ 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & -20 \end{bmatrix}$$

$$x = \begin{bmatrix} \theta \\ q \\ \alpha \\ \delta_e \\ \delta_f \end{bmatrix} \begin{matrix} \text{— pitch attitude} \\ \text{— pitch rate} \\ \text{— angle of attack} \\ \text{— elevator deflection} \\ \text{— flaperon deflection} \end{matrix}$$

$$u = \begin{bmatrix} \delta_{ec} \\ \delta_{fc} \end{bmatrix} \begin{matrix} \text{— elevator deflection command} \\ \text{— flaperon deflection command} \end{matrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 20 \end{bmatrix}$$

$$\left.\begin{matrix} \lambda_1 = -7.662 \\ \lambda_2 = 5.452 \end{matrix}\right\} \text{unstable short period mode}$$

$$\lambda_3 = 0.0 \quad \text{pitch attitude mode}$$

$$\lambda_4 = -20 \quad \text{elevator actuator mode}$$

$$\lambda_5 = -20 \quad \text{flaperon actuator mode}$$

F-16 CCV – pitch pointing

- Objectives:
  - command the pitch angle while maintaining the flight path angle
  - Stabilize short period mode, ζ=0.8, ω=7 rad/s
- Measured variables: pitch rate, normal acceleration (at pilot station), flight path angle, surface deflections

$$n_{zp} = [-0.268, 47.76, -4.56, 4.45]\begin{bmatrix} q \\ \alpha \\ \delta_e \\ \delta_f \end{bmatrix}$$

F-16 CCV – pitch pointing

Replace θ by $\gamma+\alpha$, so that θ equation is replaced by θ equation.
Choose eigenvectors in an attempt to decouple pitch rate and flight path angle.

Eigenvectors for Pitch Pointing/Vertical Translation

Desired Eigenvectors / **Achievable Eigenvectors**

$$
\begin{bmatrix} 0 \\ 1 \\ X \\ X \\ X \end{bmatrix}
\begin{bmatrix} 0 \\ X \\ 1 \\ X \\ X \end{bmatrix}
\begin{bmatrix} 1 \\ 0 \\ X \\ X \\ X \end{bmatrix}
\begin{bmatrix} X \\ X \\ X \\ 1 \\ X \end{bmatrix}
\begin{bmatrix} X \\ X \\ X \\ X \\ 1 \end{bmatrix}
\quad
\begin{matrix} \gamma \\ q \\ \alpha \\ \delta_e \\ \delta_f \end{matrix}
\quad
\begin{bmatrix} 0 \\ 1 \\ -0.9286 \\ -5.13 \\ 8.36 \end{bmatrix}
\begin{bmatrix} 0 \\ -9.5 \\ 1 \\ 0.1286 \\ -5.16 \end{bmatrix}
\begin{bmatrix} 1 \\ 0 \\ -1 \\ -2.80 \\ 3.23 \end{bmatrix}
\begin{bmatrix} -0.0057 \\ 1.07 \\ -0.0508 \\ 1 \\ 0 \end{bmatrix}
\begin{bmatrix} -0.0137 \\ 0.0601 \\ 0.0106 \\ 0 \\ 1 \end{bmatrix}
$$

Pitch Pointing/Vertical Translation Control Law

Desired Eigenvalues	Feedforward Gains	Feedback Gains
$\lambda^d_{1,2} = -5.6 \pm j4.2$ $\lambda^d_3 = -1.0$ $\lambda^d_4 = -19.0$ $\lambda^d_5 = -19.5$	$\begin{bmatrix} -2.88 & -0.367 \\ 2.02 & 4.08 \end{bmatrix}$	$\begin{array}{ccccc} q & n_z & \gamma & \delta_e & \delta_f \end{array}$ $\begin{bmatrix} -0.931 & -0.149 & -3.25 & -0.153 & 0.747 \\ 0.954 & 0.210 & 6.10 & 0.537 & -1.04 \end{bmatrix}$

F-16 CCV pitch poining

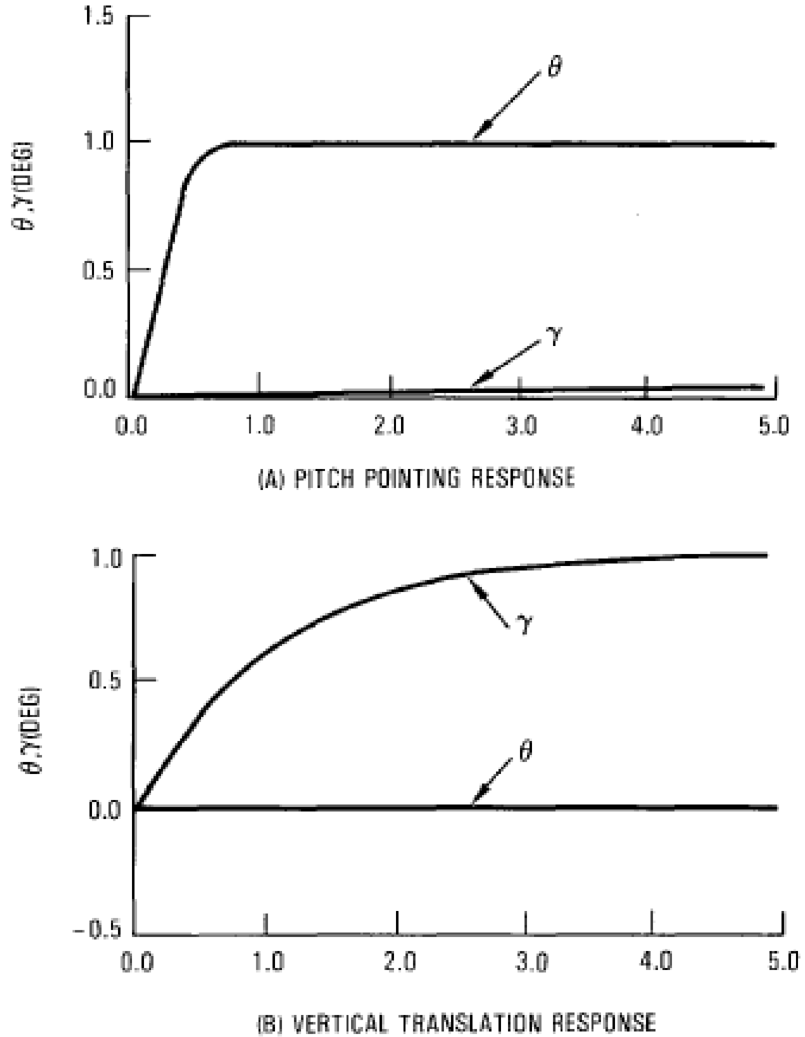


Fig. 1. Longitudinal decoupled responses.

Example L-1011, Shapiro & Chung, 1983

$$\frac{d}{dt}\begin{bmatrix}\beta\\ p\\ r\\ \phi\end{bmatrix}=\begin{bmatrix}-.117 & -.000295 & -.996 & .0386\\ -5.2 & -1.0 & .249 & 0\\ 1.54 & -.0042 & -.154 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}\beta\\ p\\ r\\ \phi\end{bmatrix}+\begin{bmatrix}.02 & 0\\ .337 & -1.12\\ -.744 & -.032\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_r\\ \delta_a\end{bmatrix}$$

	Eigenvalue	Desired Eigenvalue
Dutch Roll	-.0882 ± i 1.2695	-1.5 ± i 1.5
Roll Subsidence	-1.0855	-2.0 ± i 1.5
Spiral	-.0092	

$$\text{Dutch roll: }\begin{bmatrix}x\\ 0\\ 1\\ 0\end{bmatrix}\begin{bmatrix}1\\ 0\\ x\\ 0\end{bmatrix}\qquad \text{roll: }\begin{bmatrix}0\\ x\\ 0\\ 1\end{bmatrix}\begin{bmatrix}0\\ 1\\ 0\\ x\end{bmatrix}$$